Bubbles and Crashes: a Simulation Approach

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Dedicated to K. Vela Velupillai on his 60th Birthday

Abstract

In joint work since 2004 we have created a family of agent-based models for financial markets in which bubbles and crashes occur in imitation of real markets. The evolution of behavioral rules in these models has shed light on some possible mechanisms used by human account managers or traders. Our programming environment, NetLogo, has proved ideal for this work, and also offers a feature, HubNet, capable of extending simulations to include human as well as robot traders.

Keywords: Bubbles, crashes, agent based models, NetLogo.

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1 Introduction

Our goal in this contribution is to introduce the basic assumptions and features of our market models. Some early results of one or our experiments are given in a companion paper by Todd Feldman and ourselves. But we will begin with a brief history of the project, written by Ralph.

In 1968, after moving from the math department of Princeton University to that of the University of California at Santa Cruz, I met Dan, then a grad student. After Dan's Ph.D. and some early positions, he became professor of economics at UCLA. During the 1980s I visited frequently at UCLA, and we used to meet for lunch at the faculty club. While I had no actual involvement in mathematical economics, I nevertheless kept up on the news through Dan. Then in 1985, Dan moved to UC Santa Cruz, and we continued meeting for the occasional lunch at the faculty club.

Meanwhile, chaos theory was heating up as a new style of applied math, and the economics community was becoming curious. Richard Goodwin, as professor at Harvard, Peterhouse College (Cambridge) and Siena, had led a long-term project on non-equlibrium economics, nonlinear dynamics, and so on. He became an early adopter and harbinger of chaos theory. His 1988 lectures in Siena appeared as a book in 1990 spreading his enthusiasm.

All this led to my invitation to give the Jacob Marshak Lecture at UCLA in January, 1987. After my talk on "Nonlinear Systems, Complex Dynamics, and the Social Sciences there followed a lively Q&A, during which there were several questions from an Indian gentleman revealing a deep knowledge and understanding of dynamical systems theory. At the end, I went up to him to inquire, "Who are you?" – and thus met Vela Velupillai.

Shortly thereafter, I received an invitation to a Workshop on Mathematical Economics at the Certosa di Pontignano, Siena (May, 1991). This was the occasion of my meeting the wonderful Richard Goodwin, having a ride in his pet car, seeing Vela again, and also meeting Leonello Punzo, both former students of Goodwin. In addition, I met a group of mathematical economists reporting exciting research in bifurcations of iterated mappings of the plane, especially Laura Gardini of Urbino, with whom I did joint work in the 1990s.

After 2000, Dan began telling me of his work in evolutionary game theory, and an opportunity arose early in 2004 to jointly apply for a grant from the National Science

Foundation. The NSF program involved was aimed at new mathematical methods for the social and behavioral sciences. After reading a special issue of *Nonlinear Dynamics, Psychology, and Life Sciences* on agent-based modeling, we saw a way to apply agent-based modeling to extend evolutionary game theory. In our grant application we wrote:

> Our proposed research will extend the class of models called evolutionary games by allowing the set of strategies (actions) of each player (trader, agent) to be a continuous space, rather than just a finite set. This continues a line of study begun in joint work of Dan Friedman and Joel Yellin in 1997. The central concept of this work is the adaptive landscape.

This grant proposal was funded, and since mid 2004 we have used NetLogo – an agent-based modeling software system – to create a sequence of financial market models. Next, we will explain the basic concepts as they have evolved to date.

2 The basic model: math

Our project website, http://www.vismath.org/research/landscapedyn/, presents several models. Here we describe the simplest one, Market Model 9.0. These concepts are basic to all of our market models.

We envision a number of money market managers (typically 20 to 100 in our simulations) trading in a financial market with two kinds of assets, riskless (safe) and risky. Each manager has a choice from a continuum of strategies characterized by a nonnegative real number, u. This is her *risk parameter*, and defines the division of her portfolio between the two kinds of assets. The minimum value, $u = 0$, indicates no risk (all assets are riskless), the maximum value, $u = 1$, indicates all risk (all assets are risky), and $u > 1$ indicates leveraged investment (borrowing the safe asset).

Further, each manager has a portfolio of total worth, z, which we normally assume to be between zero and four, with $z = 1$ indicating a typical starting value. With each step, each portfolio's worth is adjusted according to its risk parameter. The safe portion u earns at rate R_0, which we have fixed at $R_0 = 0.03$, while the risky portion $(1 - u)$ earns at rate R_1, with typically $R_1 \geq R_0 \geq 0$. The manager's gross annual return is thus,

$$R_G = (1 - u)R_0 + uR_1 \tag{1}$$

Financial math leads us to pose,

$$R_1 = R_s/\bar{u}^2 + 2\dot{\bar{u}}/\bar{u} \tag{2}$$

where $R_s = R_0 + R_d$ and $R_d = 0.03$, or $R_s = 0.6$; $\bar{u}$ is the mean value of u choices for all managers; and $\dot{\bar{u}}$ is the time rate of change of $\bar{u}$. Full details may be found in (Friedman and Abraham, 2006).

Also, we assume that the gross return is decreased by a risk cost,

$$c(u) = c_2 u^2/2$$

where $c_2 = 0.02$. (In the research version of this basic model, Model 8.0, the constant c_2 may be varied by a slider.) Then the net return is,

$$R(u) = u(R_1 - R_0) - c_2 u^2/2 \tag{3}$$

Combining (1) and (2) we obtain the payoff function,

$$\phi(u, F) = u(R_s/\bar{u}^2 + 2\dot{\bar{u}}/\bar{u} - R_0) - c_2 u^2/2 \tag{4}$$

where $F(u)$ denotes the dependence of net payoff on the distribution of u choices of all managers..

The simulation proceeds in steps of discrete time intervals of size "stepsize", which the operator may choose as days, weeks, and so on. With each step, each manager's worth, z, is adjusted (depending on the stepsize) according to the net annual return $R(u)$ according to her current choice of risk, u. Additionally, her strategy choice, u, is adjusted according to the assumption of landscape dynamics, a gradient rule. That is, we assume that each manager is hill-climbing up the gradient of the payoff function (3) which depends on the current strategy choices of all managers (and their changes) through $\bar{u}$ and $\dot{\bar{u}}$ in (3).

3 The basic model: NetLogo

We now explain the graphical user interface of our simplest NetLogo model, Market 9.0, shown in Figure 1. The "population" slider (default setting, 30) determines the number of managers for the simulation. The "center" slider (default 20%) determines the mean u for the initial distribution of managers. The "setup" button creates the chosen number of managers with chosen mean u, and with random values of (u, z)

within a medium-sized rectangle in the upper half of the black graphics window. The "frequency" drop down menu (default 52, or weekly steps) determines the number of updates per year. The "go" button begins the simulation, which continues until the "go" button is pushed once more. The small triangles in the upper half of the graphics window indicate the managers, each positioned according to its (u, z) coordinates, so they are seen to move smoothly about as the simulation progresses.

The other features of the interface shown in Figure 1 are three plots and three monitors, that collectively show the position of mangers, the landscape function, the market price as a ticker-tape and as current value, the total elapsed time in years, and the current value of net risky yield, R_1.

It has been proven (see Friedman, 2006) that this model always converges to a heap of managers all in one spot, and indeed, that is we what we observe as the simulation progresses. To obtain bubbles and crashes we need a more sophisticated model, such as market Model 9.1. It implements two innovations: surprise (stochastic variations in payoff) and the c_2-dynamic (varying the c_2 coefficient in the gradient rule). All of our models run as applets from our website, and we encourage you to try them out. Our more sophisticated models have provided many insights into market forces contributing to bubbles and crashes, as reported in our articles published on our website.

4 The advanced models

In the course of our project, we made a succession of extensions to the basic model, in search of dynamical features underlying the bubble and crash behavior of real financial markets.. Three successive extensions, called Model 8.1, 8.2, and 8.3, extend the research version of the basic model, Model 8.0. In parallel, we prepared simplified models of two of these, Model 9.0 and 9.1. All of these are posted on our website with documentation. The NetLogo models posted there function as applets, that is, you may run the model within your web browser. In addition, the NetLogo models may be downloaded and run in the NetLogo programming environment, which may be freely downloaded from the NetLogo website, http://ccl.northwestern.edu/netlogo.

The first of these extensions was successful in exhibiting bubble and crash behavior, and most of our research (reported in the papers mentioned in our bibliography below) has been done with this extension, Model 8.1. The chief dynamical feature of this extension, the c_2-dynamic, has the coefficient c_2 in the the risk cost (see equation

2 above) controlled by an algorithm, rather than by a slider. Unlike the basic model, here we have endogenous perturbations affecting each manager's payoff separately, that we call *surprise*. Our model determines surprise by an Ornstein-Uhlenbeck process. Due to the occasional negative surprise, the managers accrue losses, from which we calculate (for each manager independently) a weighted sum, $\hat{L}$, with higher weights for recent losses, and declining weights for older losses. Our algorithm for the c_2-dynamic makes use of all the individual $\hat{L}$ values, combined in a global, z-weighted mean, L_m. Recall that z is a variable (for each manager) measuring the current worth of that manager's portfolio. The rule to update c_2 is $c_2 = \beta L_m$, where β as a constant.

The user interface for Model 9.1 is shown in Figure 2. Note there are several additional sliders, one of which is "beta", which sets the constant β. The others are described in the User Manual for Model 8.1.

5 Conclusion

The arrival in the 1980s of agent-based modeling in general, and NetLogo in particular, has stimulated a new wave of simulation research in economics, and more generally in the social and behavioral sciences. And it is in this context that we have situated our work performed under our recent NSF grant. However, during our work with NetLogo we discovered that it has various unique features that extend beyond the spectrum of other agent-based modeling systems.

One of these unique extras is the HubNet system. This provides NetLogo client interfaces, so that a local net of computers may share control of the graphical user interface of a simulation as it runs. Originally developed for classroom use, we have found it useful for experiments involving human subjects interacting with a market of robot managers, and in other experiments as well. This is the background for a companion article by Todd Feldman and ourselves.

We feel that this work advances the programs initiated by Richard Goodwin and his students into new levels, and that many future agent-based simulations and experiments will follow.

6 Acknowledgements

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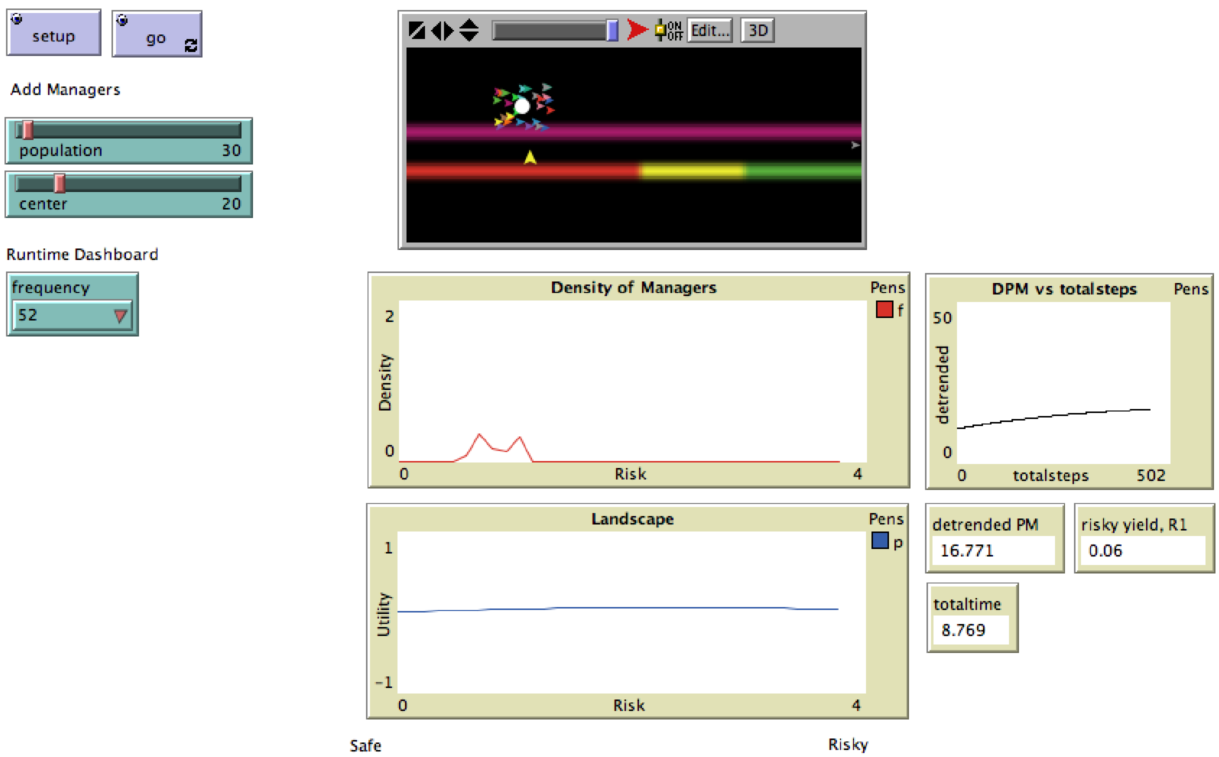

Figure 1: Interface of NetLogo Market Model 9.0.

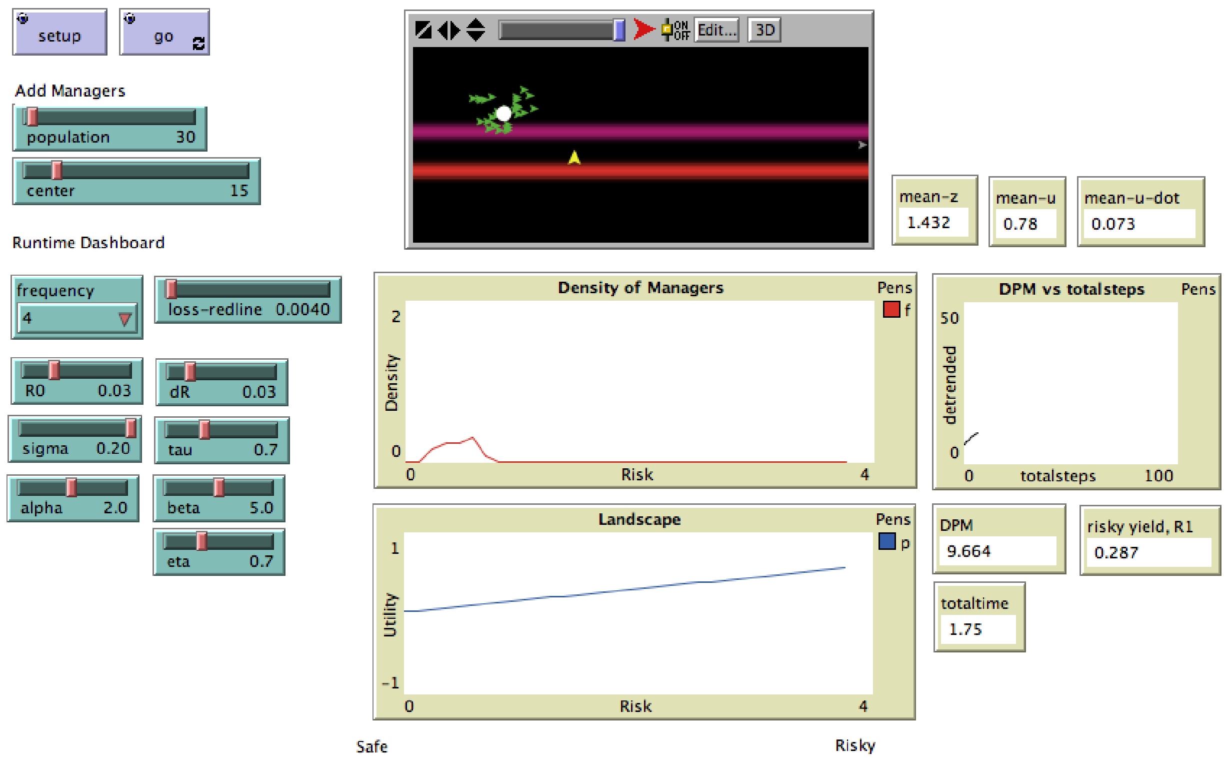

Figure 2: Interface of NetLogo Market Model 9.1.