

# Coordinate Systems for NetLogo 2D, Version 3

Ralph Abraham

February 17, 2005

## 1 The World

The two-dimensional world, in mathematical abstraction, is the Euclidean plane, with real number coordinates, say  $(u, v)$ . In computer simulation, the coordinates may take only discrete values, normally floats or doubles. Also, a bounded rectangle is defined by bounds:  $u_{min}, u_{max}, v_{min}, v_{max}$ . For example, the closed rectangle may be specified as,

$$\begin{aligned}u_{min} &\leq u \leq u_{max} \\v_{min} &\leq v \leq v_{max}\end{aligned}$$

A two-dimensional torus may be specified in the same way, but with the identification of  $u_{min}$  with  $u_{max}$ , and also,  $v_{min}$  with  $v_{max}$ .

## 2 The Screen

In NetLogo, there are two screen coordinate systems. One is an integer-valued system,  $(pxcor, pycor)$  for patches, the other float-valued,  $(xcor, ycor)$ , for turtles. Both systems are toroidal, that is, wrapped around the edges, like television.

For patches, we have bounds given by the variables, *screen-edge-x*, and *screen-edge-y*, which are attributes of the Graphics Window widget. That is,

$$\begin{aligned}-screen-edge-x &\leq pxcor \leq screen-edge-x \\-screen-edge-y &\leq pycor \leq screen-edge-y\end{aligned}$$

The default values are  $screen-edge-x = screen-edge-y = 17$ . Hence the default Graphics Window is  $17 + 1 + 17 = 35$  patches square. Because this system is toroidal, asking the patch with  $pxcor = screen-edge-x + 1$  to do something results in an action by the patch at the left edge of the screen.

Turtle coordinates are toroidal, as follows. Let us suppose, for simplicity, that the Graphics Window has the default bounds, 17. Then a turtle with  $xcor = 17.0$  appears in the center of a patch at the

right edge of the screen, while a turtle with  $xcor = -17.0$  appears in the center of a patch at the left edge of the screen.

Similarly, a turtle with  $xcor = 17.499$  appears at the right edge of the screen, but moving it microscopically to the right, to  $xcor = 17.500$ , will jump it way around to the left edge of the screen. And a turtle with  $xcor = -17.500$  appears at the exact left edge of the screen, but moving it microscopically to the left, to  $xcor = -17.501$ , will jump it way around to the right edge of the screen. The behavior of a turtle's  $ycor$  is similar. The Screen Window is closed at the West and South edges, and open at the North and East edges.

The toroidal behavior, from the mathematical point of view, is a mapping from the Euclidean plane,  $R^2$ , to the two-torus,  $T^2$ , by division, or reduction modulo the action of a group. In the following, we will indicate the Euclidean coordinates by  $(x, y)$ , so the reduction map is,

$$R^2 \rightarrow T^2; (x, y) \mapsto (xcor, ycor)$$

### 3 Affine Isomorphisms

We now wish to relate the world coordinates,  $(u, v)$ , and the screen coordinates for turtles,  $(xcor, ycor)$ . As the world coordinates are Euclidean, while the turtle screen coordinates are toroidal, we will describe directly only the affine isomorphism between Euclidean world coordinates,  $(u, v)$ , and Euclidean turtle-screen coordinates,  $(x, y)$ , or,

$$R^2 \rightarrow R^2; (u, v) \mapsto (x, y)$$

This is defined by the equations,

$$\frac{x - x_{min}}{x_{max} - x_{min}} = \frac{u - u_{min}}{u_{max} - u_{min}}$$

$$\frac{y - y_{min}}{y_{max} - y_{min}} = \frac{v - v_{min}}{v_{max} - v_{min}}$$

or, solving for  $x$  and  $y$  in terms of  $u$  and  $v$ ,

$$x = (u - u_{min})(\Delta x / \Delta u) + x_{min}$$

$$y = (v - v_{min})(\Delta y / \Delta v) + y_{min}$$

where  $\Delta x = x_{max} - x_{min}$ , and similarly for  $\Delta u$ ,  $\Delta y$ , and  $\Delta v$ . Similar equations determine  $u$  and  $v$  in terms of  $x$  and  $y$ .

## 4 An Example

Consider the two following figures. They show an exemplary Graphic Window setup in which:

- there is one row of three patches (that is,  $\text{screen-edge-y} = 0$ ;  $\text{screen-size-y} = 1$ ),
- so  $y_{min} = -0.5$ ,  $y_{max} = +0.5$ ,  $\Delta y = 1.0$ ,
- and there are three columns of patches (that is,  $\text{screen-edge-x} = 1$ ;  $\text{screen-size-x} = 3$ )
- so  $x_{min} = -1.5$ ,  $x_{max} = +1.5$ ,  $\Delta x = 3.0$ .

These patches are shown in Figure 1, with both coordinates shown, for comparison. The turtle coordinates within the Graphics Window, as fundamental domain, satisfy:

$$x_{min} = -1.5 \leq x_{cor} < 1.5 = x_{max}$$

$$y_{min} = -0.5 \leq y_{cor} < 0.5 = y_{max}$$

The turtle coordinate grid is shown in Figure 2, where the heavy boundaries are included in the fundamental domain, and the lighter boundaries not.

## 5 References

NetLogo 2.0 User Manual  
Any text on linear algebra  
Any text on computer graphics

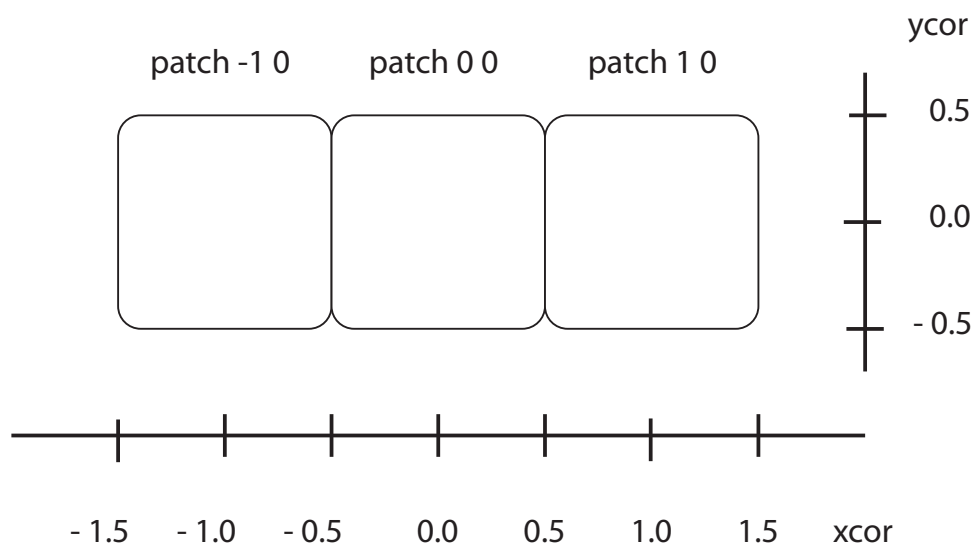


Figure 1. Patch Coordinates

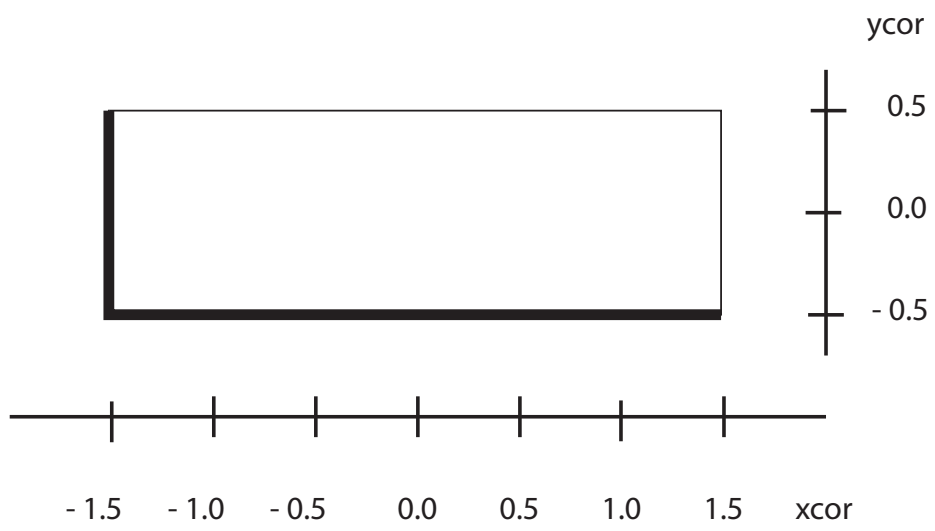


Figure 2. Turtle Coordinates, Fundamental Domain