Landscape Gradient, Veblen Model

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REVIEW

The mathematical description of the basic landscape dynamical system is as follows. Let the unit interval $A = [0, 1]$ be the space of strategies, and let $\mathcal{F}$ be the space of all cumulative distribution functions on A, with the weak-star topology. That is, $F \in \mathcal{F}$ if F is a right-continuous nondecreasing function on A with range $[0, 1]$, and countable discontinuities. Recall that $F(x)$ represents the fraction of players whose choices are no larger than x.

The *fitness* for any player choosing strategy $x \in A$, when the current state is $F \in \mathcal{F}$, is denoted by $\phi(x, F)$. The application dictates a particular fitness function $\phi : A \times \mathcal{F} \to R$. The derived function $\phi^F : A \to R$; $x \mapsto \phi(x, F)$ with F held constant, is the instantaneous *fitness landscape* for that fitness function. It depends on F only, and we think of it as a landscape, or graph in $A \times R$.

THE VEBLEN EXAMPLE

Veblen consumption illustrates several of possibilities inherent in landscape dynamics, and is interesting in its own right. Thorstein Veblen (1899) popularized the idea that some goods and services are consumed largely to gain status. Such consumption has the desired effect only to the extent that it exceeds the conspicuous consumption of other people, i.e., its utility is rank-dependent.

THE MATH

Consider a single population of consumers with identical incomes. Each consumer chooses a fraction $x \in [0, 1]$ of income to allocate to ordinary consumption, and allocates the remaining fraction $1 - x$ to rank-dependent consumption. The state is the cumulative distribution function $F(x)$ of ordinary consumption. Assume standard direct utility, $c \ln x$, from ordinary consumption x, where the parameter $c \geq 0$ represents the relative importance of ordinary consumption. Suppose that rank-dependent utility arises from envy, i.e., I compare my rank-dependent consumption $1 - x$ to everyone else's, and am unhappy to the extent that it falls short. The shortfall is $\min\{0, y - x\}$ when your rank-dependent consumption is $1 - y$. After integrating the expected shortfall by parts, one verifies that overall expected utility is

$$\phi(x, F) = c \ln x - \int_0^x F(y)dy, \tag{1}$$

with gradient

$$\phi_x = c/x - F(x). \tag{2}$$

Our NetLogo implementation closely follows the numerical equations above. We use a primitive integration in which each consumer changes her strategy according to the gradient rule, as follows. Each consumer has a numerical ID, an integer, and a position, a floating point number. For each discrete time step of size *stepsize*:

- Each consumer (in numerical order) moves $stepsize * (\phi_x)_n$ within the strategy space, where n is the index of the patch containing her current strategy, x.
- After all consumers have adjusted in this way, the density, f, distribution, F, payoff, ϕ, and gradient, ϕ_x, are recomputed.
- The time is incremented by *stepsize*.
- After each tenth step, the two plots are updated.

Given an initial distribution of consumers, upon clicking the "go" button, our program proceeds step-by-step, until stopped with another click on the "go" button.

The interface of Veblen is shown in Figure 1. On the upper left are a number of controls that allow the operator to approximate an arbitrary initial distribution of consumers, as follows.

1. There are five parallel rows, all of which are overlays of the strategy space. Using the pop-down menu labeled "puff-row", choose a row.
2. Choose a number of consumers to add to the initial distribution on the chosen row, using the slider labeled "population".
3. Choose a subinterval of the strategy space in which to randomly locate them, using the sliders labeled "center" and "width". Both are calibrated in percent of the unit interval.
4. Push the button labeled "setup".
5. Repeat 1-2-3, then push "puff" to add another square wave of consumers, as many times as you wish.

Mathematically, all consumers are on the same strategy space, the gray row. But the consumers are shown, as colored triangles, in five puff-rows on the screen. The two plots show the density and the landscape for the initial distribution. The color bar at the bottom of the screen shows where the slope is positive (magenta), zero (yellow), and negative (cyan).

On the lower left are controls for the step-by-step integration. Set "amp" (the parameter "c" in the model) and "stepsize" (proportion of slope for a consumer to move). Then push "step" for one step of integration, or "go" for a sequence of steps. These continue until "go" is pushed again. The current step number and time are shown in the monitors labeled "totalsteps" and "totaltime". A run may be continued by again pushing "go". (The command center is for NetLogo experts.)

END: Veblen notes